CS 3313
Foundations of Computing:

Lab 3: NFA and Regular Expessions Review

Outline

- Building NFAs
- NFA to DFA Conversion
- Regular Expressions
- NFA to Regular Expressions Conversion

NFA vs. DFA

NFAs have 2 critical differences from DFAs

- Allow state-to-state transitions on empty string input ϵ
  - These transitions are done spontaneously, without looking at the input string.
- Allow more than one outgoing transition on same symbol
  - Allows the NFA to choose which path to take
  - Still need to verify that path taken leads to an accept state

Important: Still need to make sure that only strings in desired language are accepted

Exercise 1: work in groups

- Provide an NFA M that accepts the language L over alphabet {0,1,2} where L = { w | (a) w has two consecutive 0's or (b) w has a substring 101 and ends with two 2's }
  - Ex: 0120012 is in L 0102101222 is in L

    02010220 is not in L

Property (a): build NFA M1 that recognizes substring 00

Property (b): build NFA M2 that recognizes two properties in sequence – substring 101 and then ends with two 2's.

Note: We built a reg. exp. for this in class

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Converting NFA to DFA

- We proved in class that NFAs and DFAs recognize the same languages
- So, for every NFA N, we can construct equivalent DFA M
- In class, we gave a procedure for converting an NFA to an equivalent DFA

Let's review

NFA $N=(Q,\Sigma,\delta,q_0,F)$

- Q – set of states
- Σ – alphabet
- q_0 – start state
- F – accept state(s)
- δ – transition function

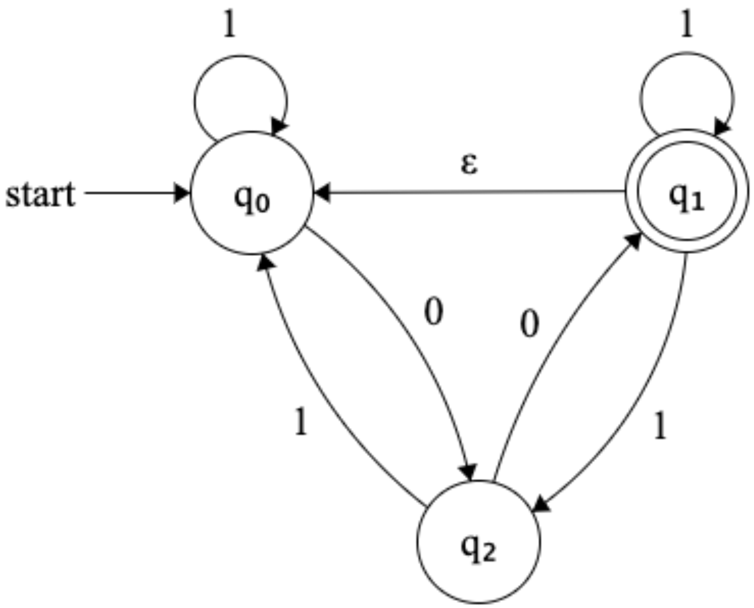
DFA $M= (Q',\Sigma',\delta',q_0',F')$

- Q' = P(Q) – powerset of Q
- $\Sigma' = \Sigma$
- q_0' = E(q) – set of states reachable from q via ϵ edges
- F' = Set of nodes in Q' that contain an accept state from Q
- δ' = Use the "finger trick":

  i.e., set of all possible states that can be reached from current set $q' \in Q'$

Exercise: NFA to DFA – Work in groups

Construct a DFA equivalent to the following NFA



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Languages Associated with Regular Expressions

- A regular expression (RE) **r** denotes a language **L(r)**
- Basis: Assuming that r_1 and r_2 are regular expressions:
  1. The regular expression Ø denotes the empty set
  2. The regular expression ϵ denotes the set { ϵ }
  3. For any a in the alphabet, the regular expression **a** denotes the set { a }
- Inductive step: if r_1 and r_2 are regular expressions, denoting languages $L(r_1)$ and $L(r_2)$ respectively, then
  1. $r_1 \cup r_2$ is a RE denoting the language $L(r_1) \cup L(r_2)$
  2. $r_1 r_2$ is a RE denoting the language $L(r_1) \circ L(r_2)$
  3. (r_1) is a RE denoting the language $L(r_1)$
  4. $(r_1)^*$ is a RE denoting the language $(L(r_1))^*$

Deriving Regular Expressions

- ”map” property in the language to a Reg.Expr. Pattern
- Break down the properties into union, concatenation, star
- Start with smallest reg expression (simplest property)

- Ex: all strings in alphabet {a,b} = $(a \cup b)^*$
- Two consecutive a’s = aa
- Ends with a pattern aba: $(a \cup b)^* aba$
- ….

Regular Expressions - Examples

1. L_1= { all strings over alphabet {a,b,c} that contain no more than three a's }
2. L_2 = { all binary strings ending in 01 }

Regular expressions Examples

1. L_1= { all strings over alphabet {a,b,c} that contain no more than three a's }

- Can contain zero a's or 1 a or 2 a's or 3 a's; and can have any number of b,c before and after
- $=(b\cup c)^*\cup\big((b\cup c)^*a(b\cup c)^*\big)\cup\big((b\cup c)^*a(b\cup c)^*a(b\cup c)^*\big)\cup((b\cup c)^*a(b\cup c)^*a(b\cup c)^*a(b\cup c)^*)$

2. L_2 = = { all binary strings ending in 01 }
   - Any string w in {0,1}* followed by 01 = $(0\cup 1)^*01$

Exercise 3: Regular Expressions – Work in groups

L_3 = { all binary strings that do not end in 01 }

- Hint: you can have strings of length 0 or length 1 – what are they ?
- If string has length two or more, then what substrings can it end in (i.e., what can the rightmost two symbols be ?)
  - It cannot end in 01

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DFA/NFA to Regular Expression

- We outlined a procedure in the lecture based on state elimination
  - Can be tedious to do by hand for a small-ish DFA/NFA

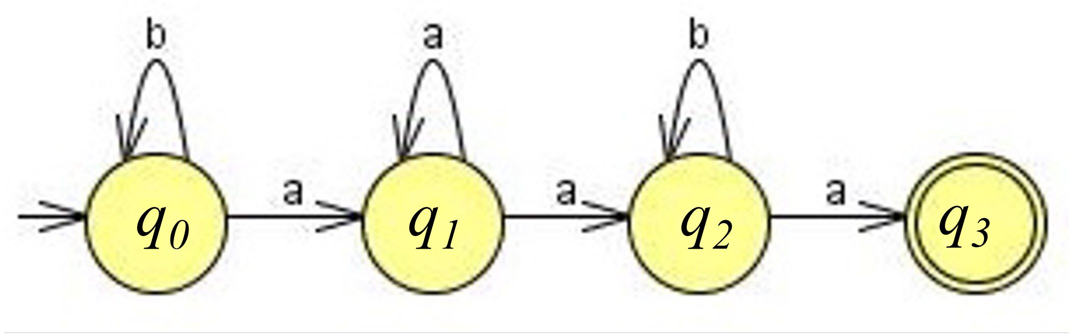
- Alternate approach: by examining the automaton and figuring out the expressions for paths to a final state
  - This works well for simple DFA/NFA, but may be hard for more complicated examples

DFA/NFA to Regular Expression

- language accepted by a DFA/NFA = { w | there is a path labelled w from start state to a final state}
- To find regular expression for the language accepted by a DFA/NFA, find the labels (and reg. expr.) of the paths from start state to each final state
  - Concatenate labels on the path – the label is the regular expression
    - Concatenate labels on the subpaths
  - If we have two choices of paths with labels w_1 and w_2 then "or" the paths to get w_1+w_2
  - If there is a cycle, with path labelled *w*, then *w**

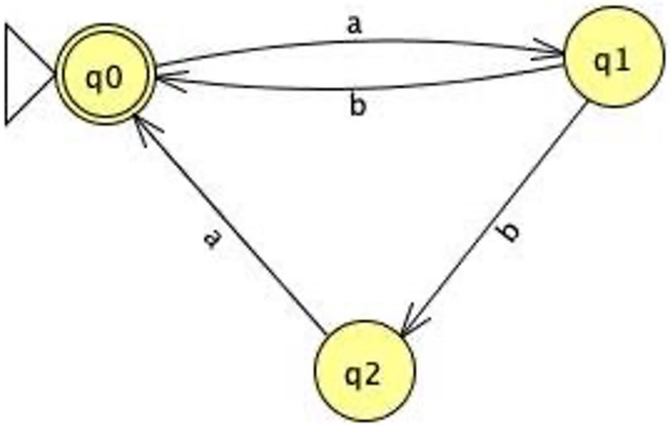
DFA to Reg.Expression – Example 1

- Find expression for paths from q_0 to q_3:
  - Paths from q_0 to q_1 followed by q_1 to q_2 followed by q_2 to q_3
- b* a followed by a*a followed by b*a
- Reg expr= $b^*a\ a^*\ a\ b^*\ a$



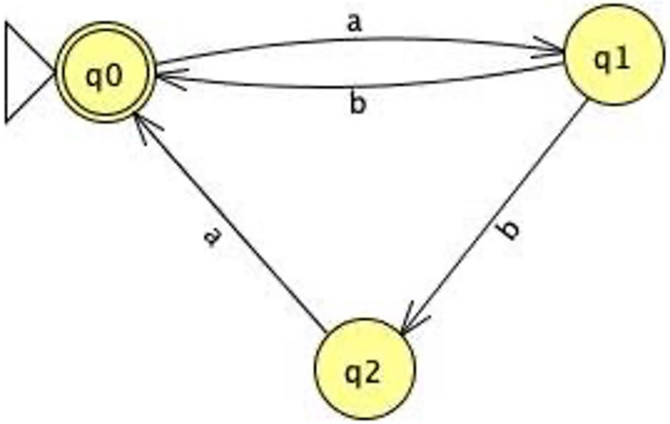
Automaton to Reg. Expression – Example 2

- Find expression for all paths from start state to a final state
- Example: paths from q_0 to q_0
  - q_0 to q_1 to q_0 =
  - q_0 to q_1 to q_2 to q_0 =
  - But: can repeat cycle from q_0 to q_0
  - q_0 to itself on empty string λ
- Therefore: *Reg. Exp.* =



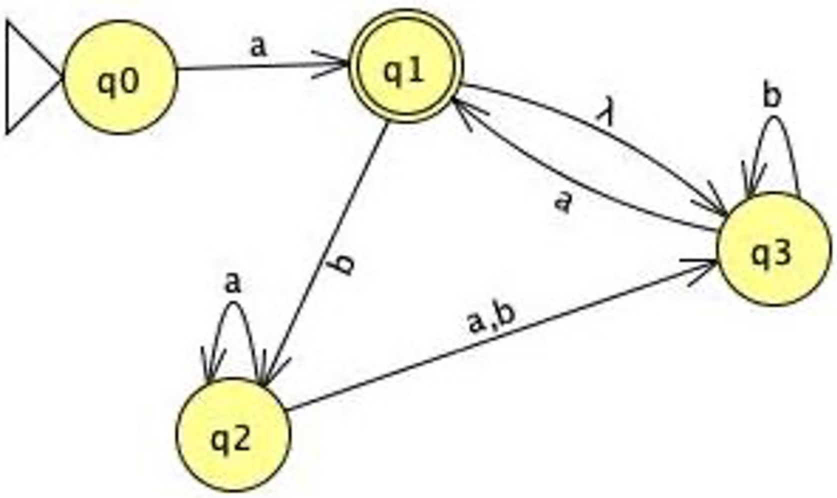
Automaton to Reg. Expression – Example 2

- Find expression for all paths from start state to a final state
- Example: paths from q_0 to q_0
  - q_0 to q_1 to q_0 = (ab)
  - q_0 to q_1 to q_2 to q_0 = (aba)
  - But: can repeat cycle from q_0 to q_0
  - q_0 to itself on empty string λ
- Therefore: $Reg.\ Exp. = (ab \cup aba)^*$



NFA to Reg. Expression – Example 3

- Direct edge label a from start to the final state q_1
- Cycles/path from q_1 to q_1 : consider the two paths –
  - either utilization the ϵ : $\epsilon b^* a = (b^* a)$
  - or not $(ba^*(a \cup b)b^* a)$
- Therefore cycle is: $\left((ba^*(a \cup b)b^* a) \cup (b^* a)\right)^*$
- Therefore reg. expr. Is $a\left((ba^*(a \cup b)b^* a) \cup a(b^* a)\right)^*$



Exercise 4: DFA to Reg. Exp. – Work in groups