Foundations of Computing

Lecture 11

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Outline

Lecture 10 Review

- CFG == PDA
  - Construct PDA from CFG
  - Construct CFG from PDA
- CFG Pumping Lemma

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Today

- Midterm review

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The CFG Pumping Lemma

Theorem

If L is a CFL, then there exists a pumping length p s.t. for any $s \in L$, with $|s| \geq p$, s can be divided into 5 pieces $s = uvxyz$ satisfying:

1. For each $i \geq 0$, $uv^i xy^i z \in L$
2. $|vy| > 0$
3. $|vxy| \leq p$

- Last week we saw how to use this to prove languages not context-free

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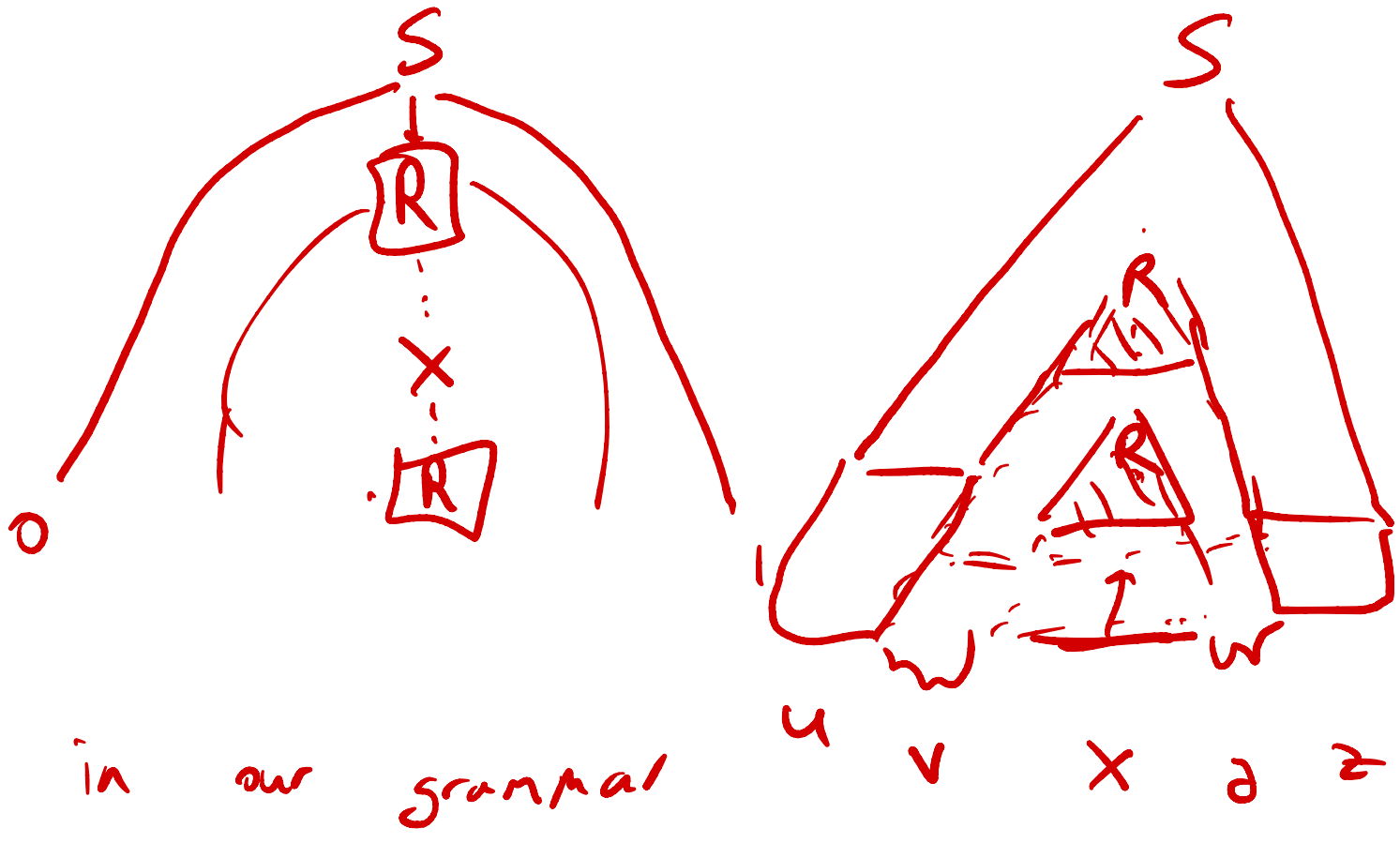
- Last week we saw how to use this to prove languages not context-free
- But, we did not explain why this lemma is true

Proving the CFG Pumping Lemma (Intuition)



p Ruby in our grammar

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1. DFA
   - Know what it means for a DFA to accept a string
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   - Recall closure properties of regular languages (complement, union, intersection, concatenation, $*$ closure)

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   - NFA to DFA using the finger method

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Topics

1. Assume L is reg.
2. Observe that $0^* 1^*$ is reg.

3. Since reg. Lang. are closed under $\cap$

$\Rightarrow L_1$ is regular

Contradiction!

5. Regular Expressions
   - Be able to build an RE for a language
   - RE to NFA
   - NFA to RE
6. Regular Language Pumping Lemma
   - Remember statement as sequence of quantifiers
   - Understand why it is true (state of NFA must repeat)
   - Understand how to use it.
   - Also know how to prove languages not regular using closure properties

$L: \{ w \mid \#0's = \#1's \}$ $\quad L_1 = L \cap 0^* 1^*$

$L_1: \{ w \mid 0^n 1^n \}$ – not regular

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   - Remember what this means

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   - Remember what this means
   - Be able to construct one from language description
   - Remember what a derivation is and what a parse tree is
   - PDA == CFG (at a high level)
9. CFL pumping lemma
   - There will not be any questions on the CFL pumping lemma on the exam
   - But, there will be on the next homework

Exam Format

- 7 questions – most have multiple parts
- Covers most of the material outlined above
- 2 questions requiring proofs, the rest are more constructive
- Some yes/no questions

Don't Forget

- Exam is in class on Thursday 11:10-12:25, don't be late!
- You can bring two 8.5×11 piece of paper

Any Questions?

if L is regular

$\exists p$ s.t. $\forall w \in L$ s.t. $|w| > p$

$\exists$ partition $w = xyz$ s.t.

1) $|y| > 0$
2) $|xy| < p$
3) $\forall i \geq 0 \quad xy^i z \in L$

Any Questions?

Assume L is regular

$\forall p \; \exists w \in L, |w| \geq p$ s.t.

$\forall$ partitions $w = xyz$

$\exists i$ s.t. $xy^i z \notin L$

Any Questions?

$L = 0^n 1^{2n}$

$S \to 0S11 \mid \epsilon$

Any Questions?

$L = 0^{n-3}\,1^n$ $\quad L_1 = \{000\}$

$L_1 = \{w \mid \text{s.t. } w = 0^n 1^n \text{ for } n \geq 1\}$

$L_1 = 000 \,\|\, 0^{n-3}\,1^n$

$L_1 = L_2 \,\|\, L$

Any Questions?