Foundations of Computing

Lecture 14

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March 5, 2024

Outline

1. Lecture 13 Review
2. Specification of a Turing Machine
3. Decidable and Turing-recognizable Languages
4. Languages With Machines as Input
5. Preliminaries – Countable and Uncountable Sets

Lecture 13 Review

- More Turing Machines
- Turing Machine Variants
  - Multi-tape Turing Machines
  - Non-deterministic Turing Machines

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Important TM Notation / Observations

- TM always takes a string as input
  - Sometimes we want to talk about a TM taking another type of input (e.g., a graph, a FA, a TM)
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- Can give a machine as an input to another machine
  - All machines we have seen can be written as finite tuples, e.g. $(Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$
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  - A TM that takes ANY TM as input and runs it is called a *universal TM*

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- Are there some problems that inherently do not have any algorithmic solution?

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Observation

Every Decidable language is also Turing-recognizable, but the reverse direction may not be true.

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A Second Question

What about Turing-unrecognizable languages?

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Taking Machines as Input

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- So, can give a description of a machine M to another machine M'
- Today, we will talk about TM's that run another machine M'

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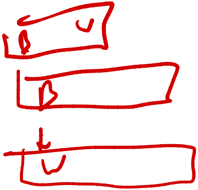

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2. If simulation ends in an accept, then accept. If it ends in a non-accepting state, then reject

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2. Run TM from previous slide on input $\langle C, w\rangle$
3. Output what this TM outputs

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Algorithm to decide A_{REX}:
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1. Convert R into equivalent DFA M'
2. Run decider on M', w
3. Output whatever M' outputs

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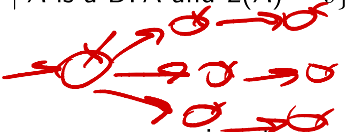

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Corollary

Every CFL is decidable

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Is A_{TM} Turing-recognizable?

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The HALTING Problem

- This is known as the HALTING problem
- We will prove that it is undecidable

Relationships Among Language Classes