Foundations of Computing

Lecture 20

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April 4, 2024

Outline

1. Lecture 19 Review
2. Verifying vs. Deciding
3. Nondeterministic Polynomial Time

Lecture 19 Review

- Polynomial Time Computation
- The Complexity Class $\mathcal{P}$

$$\mathcal{P} = \bigcup_k TIME(n^k)$$

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3. Nondeterministic Polynomial Time

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  - Pretty much everything you studied in algorithms class
- But, some problems have resisted our efforts to find efficient algorithms
- Today we will study one important class of such problems

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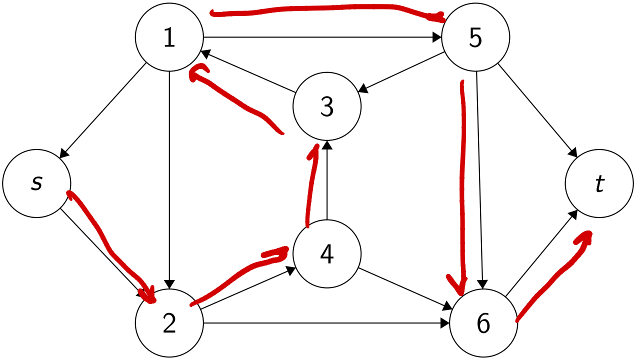

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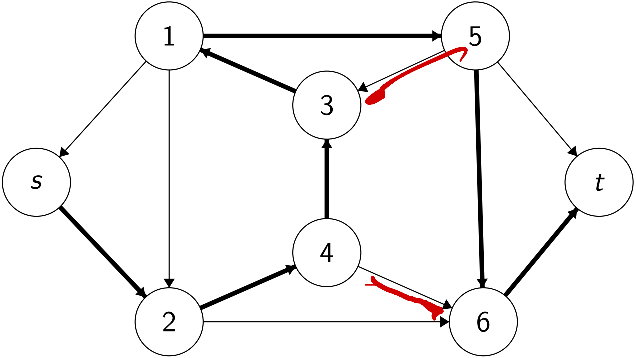

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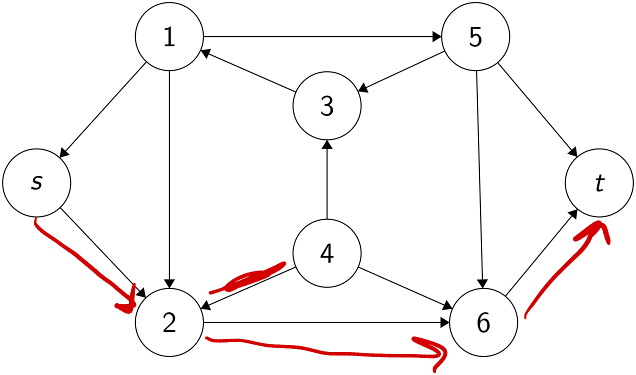
But, not every graph has a Hamiltonian Path.



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$n!$

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Polynomial Verifiability

However, given a path from s to t, can easily verify whether it is Hamiltonian in polynomial time.

Satisfiability

Boolean Formula

A Boolean formula is an expression inolving Boolean variables and logic operations AND ($\wedge$), OR ($\vee$), and NOT ($\neg$ or $\overline{x}$).

$$\phi = (\overline{x} \wedge y) \vee (x \wedge \overline{z})$$

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- Not all formulas are satisfiable

$$\phi' = (\overline{x} \wedge y) \wedge (x \wedge \overline{z})$$

if $x = 0$

if $x = 1$

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Polynomial Verifiability

However, given an assignment (i.e., values for all the variables), can easily verify whether ϕ is satisfied by this assignment in polynomial time.

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$|w| \leq poly(|x|)$

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- String w is called a witness that $x \in L$

SAT   x = formula   w = assignment

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$\exists$ D s.t. D decides L in poly-time

V(x, w)

1. Run D(x)
   output that

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- $\mathcal{NP}$ is the class of problems where you can verify a solution in poly-time
- Question: $\mathcal{P} \stackrel{?}{=} \mathcal{NP}$

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- ($\Leftarrow$) A verifier simulates the NTM by using the accepting branch as the witness

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     3. Accept if V accepts and reject otherwise

→ if $x \in L$, $\exists w$ s.t. $V(x, w) = 1$

if $x \notin L$, $\forall w$ $V(x, w) = 0$

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The Class $\mathcal{NP}$

We can define the class of languages decided by poly-time NTMs

Definition

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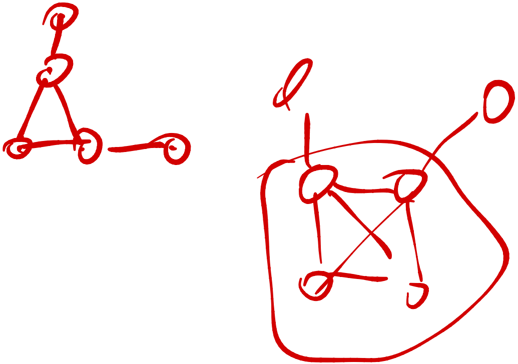
$$\mathcal{NP} = \bigcup_k NTIME(n^k)$$

Problems in $\mathcal{NP}$ – Example 1

Clique

A clique in and undirected graph is a subset of nodes s.t. every two nodes are connected by an edge. A k-clique is a clique containing k nodes



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$$CLIQUE = \{\langle G, k\rangle \mid G \text{ is an undirected graph with a } k\text{-clique}\}$$

Problems in $\mathcal{NP}$ – Example 2

Subset Sum

Given a collection of integers $\{x_1, \ldots, x_k\}$ is there a subset of them that adds up to k?

Problems in $\mathcal{NP}$ – Example 2

Subset Sum

Given a collection of integers $\{x_1, \dots, x_k\}$ is there a subset of them that adds up to t?

$$SUBSET-SUM = \{\langle S, t\rangle \mid S = \{x_1, \dots, x_k\} \text{ and for some } \{y_1, \dots, y_l\} \subseteq \{x_1, \dots, x_k\}, \sum y_i = t\}$$

The Million Dollar Question

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- Is it easier to verify a solution than to find that solution?
- This is the biggest open question in complexity theory

Let's Try to Answer It

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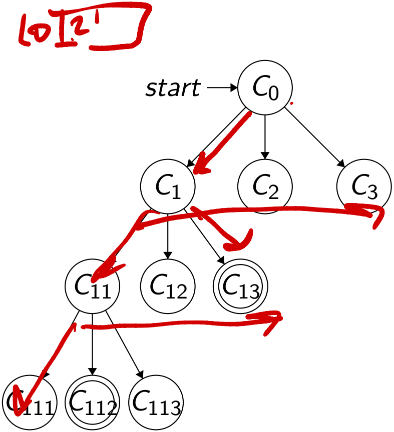
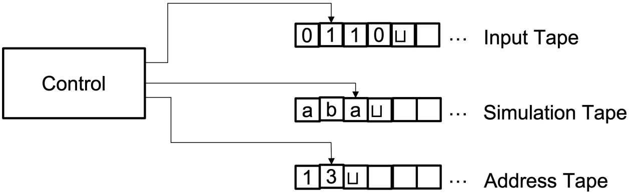


- Recall that an execution of a DTM is a sequence of configurations
- Execution of an NTM is a tree of configurations (branches correspond to non-deterministic choices)
- If any node in the tree is an accept node, the NTM accepts
- To simulate an NTM by a DTM, need to try all configurations in the tree to see if we find an accepting one

Simulating NTM on a 3-tape DTM



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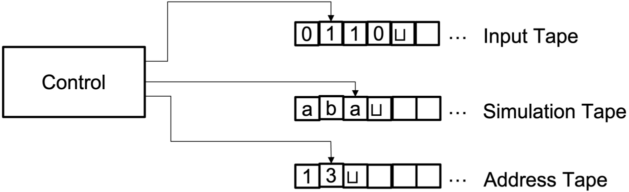

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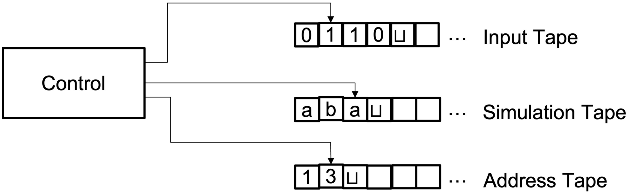

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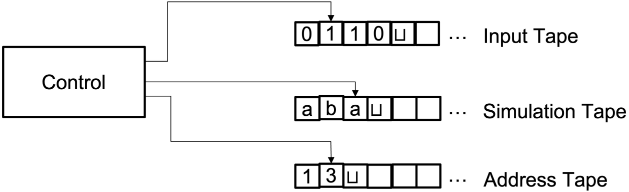

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3. Address tape – use to store which nondeterministic branch you are on

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1. Start with input w on tape 1, and tapes 2,3 empty

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- Above algorithm tries all possible values for these branches: $2^{O(t(n))}$

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What's the Problem?

- NTM running in time $t(n)$, makes $O(t(n))$ non-deterministic choices
- Above algorithm tries all possible values for these branches: $2^{O(t(n))}$
- Resulting DTM runs in exponential time

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- We will show this using reductions – Yay!