Foundations of Computing

Lecture 6

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February 1, 2024

Outline

Lecture 5 Review

- Regular expressions
- Equivalence of regular expressions and NFAs/DFAs

Quiz Solutions

For each of the following languages over $\Sigma = \{a, b\}$, give two strings that are in the language and two strings not in the language.

1. $a^* \cup b^*$ aaa aabb

2. $(aa \cup bb)^*$ aabbaa b

3. $\Sigma^* a \Sigma^* b \Sigma^* a \Sigma^*$ aaabaaa

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What We Know So Far

The following four things are equivalent:

1. Regular languages
2. Languages recognized by a DFA
3. Languages recognized by an NFA
4. Languages described by a regular expression

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Are all languages regular?

Today we will see that there are languages that are not regular

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This means that:

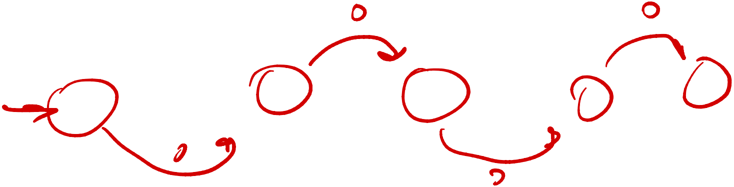
- The number of states is fixed independently of the input size
- An automaton must be able to process strings w s.t. $|w| > |Q|$
- Thus, a finite automaton cannot store its whole input

A Nonregular Language

Consider the following language:

$$L = \{0^n 1^n | n \geq 0\}$$

Let's try to build a DFA for L:



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The Problem

We need to count the number of 0s, but this is unbounded so can't have a state for each value

The Need for a Proof

What we just saw

Intuition: An NFA/DFA cannot count unbounded inputs

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Why isn't this a proof?

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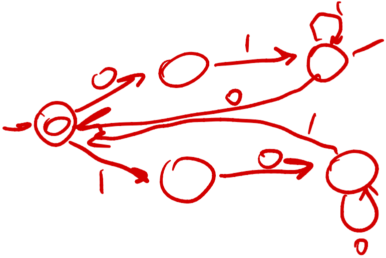
1001

Consider the following language:

$L = \{w | w \text{ has an equal number of occurrences of 01 and 10 as substrings}\}$



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General Proof Structure

$L = 0^n 1^n$

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2. Pick a string $w \in L$
3. Show that if $M(w) = 1$ then there exists a string $w' \notin L$ s.t. $M(w') = 1$
4. Conclude that L is not regular since any M that accepts all strings in L must also accept strings not in L

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$\exists p \; \forall w \in L \; \exists$ partition s.t. $\forall i \quad xy^iz \in L$

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Next steps:

1. Prove the pumping lemma
2. Show how to use the pumping lemma to prove languages nonregular

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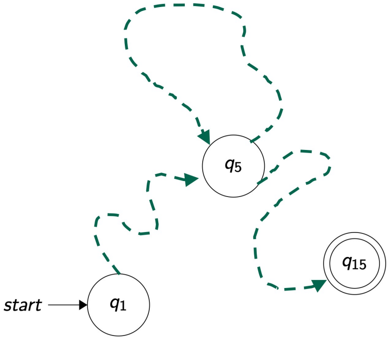
  - Since $n + 1 > p$, there must be some state that is visited twice



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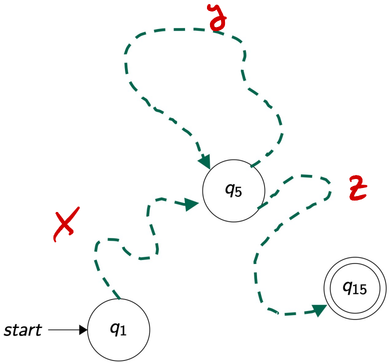

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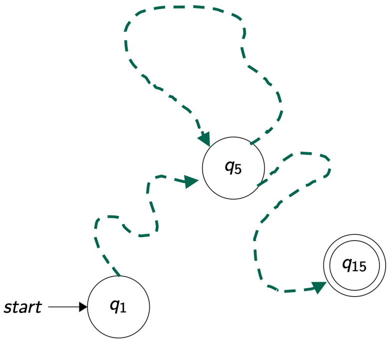

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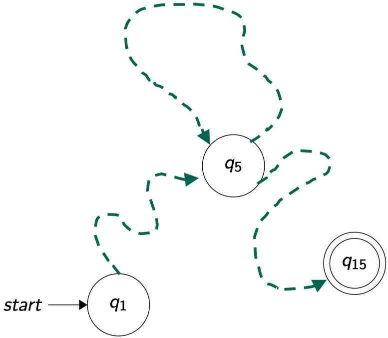

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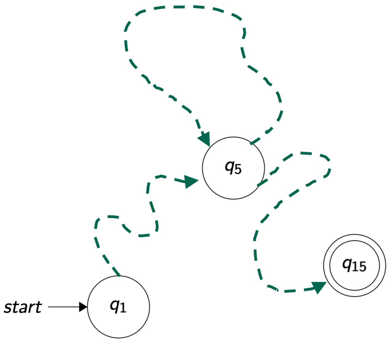

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3. $|xy| \leq p$
   Proof: if q_5 is the first repetition in $M(w)$, then this repetition must occur in the first $p + 1$ states, hence $|xy| \leq p$

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5. Contradiction!!!

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   - But, this means that y must be all 0s

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4. Problem: If $y = 0^m1^m$, then w can be pumped – no contradiction
5. Solution: Use condition that $|xy| \leq p$
   - Since $w = 0^p1^p$ and $|xy| \leq p$, we know that y must be in first p symbols
   - But, this means that y must be all 0s
6. Complete proof by considering all possible values for y
   - y consists of only 0s – then $xyyz$ has more 0s than 1s, so $w \notin L$

Example 2

Consider $L = \{w | w$ has an equal number of 0s and 1s$\}$, prove L is not regular

Proof:

1. Assume L is regular, and let p be the pumping length this implies
2. Choose $w = 0^p 1^p$
3. By pumping lemma, $w = xyz$ s.t. $xy^i z \in L$
4. Problem: If $y = 0^m 1^m$, then w can be pumped – no contradiction
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   - Since $w = 0^p 1^p$ and $|xy| \leq p$, we know that y must be in first p symbols
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   - y consists of only 0s – then $xyyz$ has more 0s than 1s, so $w \notin L$
7. Contradiction – hence, L is not regular

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4. Since regular languages are closed under $\cap$, if L is regular then L_1 must be regular
5. Since we know L_1 is nonregular, this means that L must be nonregular

Exercise

Prove that the following language is nonregular:

$$L = \{0^i 1^j 2^i 3^j | i, j > 0\}$$

What's Next?

- We will get plenty of practice with proving languages nonregular
- We will add (a small amount of) memory to our machines to recognize a richer class of languages